CHEMICAL ENGINEERING (GATE & PSUs)

Postal Correspondence

STUDY MATERIAL (Handwritten Notes)

By Ajay Sir

MECHANICAL OPERATION

GATE-2022 Syllabus: Chemical Engineering: MO

Particle size and shape, particle size distribution, size reduction and classification of solid particles; free and hindered settling; centrifuge and cyclones; thickening and classification, filtration, agitation and mixing; conveying of solids, flow past immersed bodies including packed and fluidized beds.

MECHANICAL OPERATION COURSE CONTENT

1. **Introduction**
2. **Characterization of solid particle**
3. **Size reduction of solids**
4. **Mixing equipment**
5. **Sedimentation theory (Solid-Liquid separation)**
6. **Solid-Gas separation**
7. **Fluidized bed and fluidization**
8. **Filtration**
9. **Solid-Solid separation**
10. **Transportation of solids**

Note for Student:

1. Full GATE Syllabus covers in Notes.

2. Total number of pages in MO Notes = 150 Pages

3. No. of Questions solved in Notes = 75+ Questions

(GATE PYQs & other good quality question)

Mechanical operation :→ A ⟶ P

Chemical Engineering

- **unit operation** (physical changes)
  - **Mechanical operation**
    - properties of solids
    - size reduction
    - size separation
    - Transportation of solids
    - Mixing etc.
  - Fluid Flow
  - Heat transfer
  - Mass transfer
- **unit process** (chemical changes)
  - oxidation
  - Hydration
  - Hydrogenation
  - Halogenation
  - Nitration

* Mechanical operation :-

"Mechanical operations are the unit operations of chemical engineering in which mechanical forces, either small or large, are involved for the processing and handling of solids as such and solids present in other phases".

* MECHANICAL OPERATION :-

The physical operation/changes carried out in unit operation are done using some basic equipment like grinders, filters, etc.

Characterisation of solid particles :-

→ size and shape are inter-related since in order to define a size, one has to make some assumption about shape.

→ for some regular shapes, there is a single measurement which completely defines the particle. eg:- sphere and cube

→ for other regular shapes, more than one measurement which defines the particle eg:- cylinders and cuboids.

→ for irregular shapes, some typical dimensions must be defined ex:- sphericity (ϕ)

* **Size :-** (particle size) : size is the linear dimension of the particle. for irregular particles, the size may be found as the average of the shortest and the longest dimension of the particle or as the second largest dimension.

* particle size are expressed in different units

	Particle size	units
I)	Coarse	inches or millimeters (in or mm)
II)	fine	screen size
III)	very fine	Micrometers or nanometers (μm or nm)
IV)	ultra fine	surface area per unit mass (m^2/gm)

* Particle size can be measured using of measuring techniques :-

1) Screening (for particles of size > 50 μm)
2) Sedimentation (size range 1 - 100 μm)
3) Elutriation (size range 5 - 100 μm)

Particle shape :-

The shape of an individual particle is commonly expressed in terms of the sphericity (ϕ_s), which is independent of particle size,

* for a spherical particle of diameter D_p, $\boxed{\phi_s = 1}$

* for a non-spherical particle size, the sphericity is defined

$$\left\{ \phi_s = \frac{\text{surface Area of sphere having same volume that of particle}}{\text{surface Area of particle}} \right\}$$

$$\boxed{\phi_s = \frac{6 / D_p}{S_p / V_p}}$$

Where ϕ_s = sphericity

D_p = equivalent diameter or nominal diameter

S_p = surface Area of one particle

V_p = volume of one particle

* Equivalent diameter :- It can be defined as the diameter of a sphere of equal volume.

Ques 1) find the sphericity of a cube having dimension a

Sol) $\left(\phi_s = \frac{\text{S.A of sphere}}{\text{S.A of particle}}\right) = \frac{4\pi R^2}{6a^2}$

volume of sphere = volume of particle

$\frac{4}{3}\pi R^3 = a^3$

$R = \left(\frac{3}{4\pi}\right)^{1/3} a$

$\phi_s = \frac{4\pi\left(\frac{3}{4\pi}\right)^{2/3} a^2}{6a^2} = 0.805$

$(\phi_s)_{cube} = 0.805$ Answer

Ques 2) find the sphericity of cylinder particle of diameter 3 mm of height 3 mm.

Sol) $\left(\phi_s = \frac{\text{S.A of sphere}}{\text{S.A of Particle}}\right) = \frac{4\pi R^2}{2\pi r(r+H)}$

S.A of cylinder = $2\pi r(r+H)$

volume of cylinder = volume of cylinder

$\frac{4}{3}\pi R_s^3 = \pi R^2 H$

$R_s^3 = \frac{3}{4} R^2 H = \frac{3}{4}\left(\frac{3}{2}\times10^{-3}\right)^2 (3\times10^{-3})$

$R_s = 1.717\times10^{-3}$

$\phi_s = \frac{4\pi(1.717\times10^{-3})^2}{2\pi\left(\frac{3}{2}\times10^{-3}\right)\left[\frac{0.003}{2} + 0.003\right]} = 0.873$

ϕ_s Answer

Que 3) find the sphericity for a cuboid of (10 × 5 × 1 cm).

Sol →

$$\left(\phi_s = \frac{\text{S.A of sphere}}{\text{S.A of cuboid}}\right) = \frac{4\pi R^2}{2(lb + bh + hl)}$$

volume of sphere = volume of cuboid

$$\frac{4}{3}\pi R^3 = 10 \times 5 \times 1$$

$$R = 2.285 \text{ m}$$

$$\phi_s = \frac{4\pi(2.285)^2}{2[(10\times5) + (5\times1) + (1\times10)]} = 0.504$$

$\phi_s = 0.504$ Answer

* Imp. **sphericity (ϕ_s) :-**

(ϕ_s) sphere >	cylinder >	Hemisphere >	cube >	cone >	coarse Particle
1	0.873	0.84	0.81	0.79	0.6 to 0.78

* surface Area of sphere is minimum

$$\boxed{\phi_s \le 1} \quad \text{(always)}$$

$$\phi_s = \frac{\text{S.A of sphere}}{\text{S.A of particle}},$$

sphere volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d_p}{2}\right)^3$

$$V_p = \frac{1}{6}\pi d_p^3$$

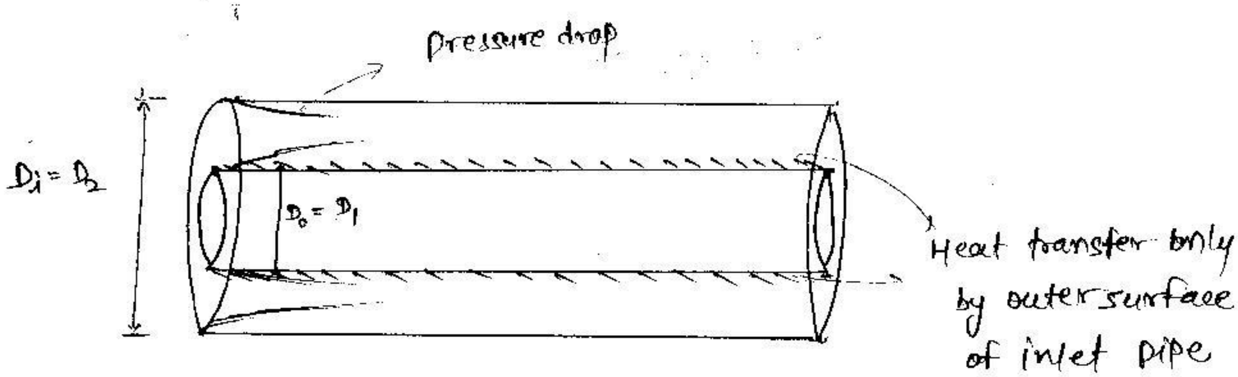
$$6V_p = \pi d_p^3 \Rightarrow \pi d_p^2 = 6V_p/d_p$$

surface Area of sphere = $4\pi r^2 = 4\pi \frac{d_p^2}{4}$

$$\pi d_p^2 = 6V_p/d_p$$

$$\phi_s = \frac{\pi d_p^2}{S_p} \Rightarrow \frac{6V_p/d_p}{S_p} \Rightarrow \left\{\phi_s = \frac{6/d_p}{S_p/V_p}\right\}$$

* Equivalent diameter :→ (D_{eq})



$$\left[D_{eq} = 4 \times \frac{\text{flow Area}}{\text{wetted perimeter}} \right]$$

$$\left(D_{eq} = 4 \frac{A}{P} \right)$$

(I) **Basis on pressure drop :-** $[D_{eq} = 4 r_H]$

r_H = Hydraulic diameter $= \frac{A}{P}$

$\{\text{wetted perimeter} = \pi (D_1 + D_2)\}$

$$D_{eq} = 4 \cdot \frac{A}{P} = 4 \frac{\left\{ \frac{\pi}{4} D_2^2 - \frac{\pi}{4} D_1^2 \right\}}{\pi (D_1 + D_2)} = \frac{\pi (D_2 - D_1)(D_2 + D_1)}{\pi (D_1 + D_2)}$$

$$\boxed{D_{eq} = D_2 - D_1} \quad ! \quad \text{or} \quad (D_{eq} = D_o - D_i)$$

(II) **Basis on Heat transfer :-** $\{\text{wetted perimeter} = \pi D_1\}$

$$D_{eq} = 4 \frac{A}{P} = \frac{4 \left\{ \frac{\pi}{4} D_2^2 - \frac{\pi}{4} D_1^2 \right\}}{\pi D_1} = \frac{D_2^2 - D_1^2}{D_1}$$

$$\boxed{D_{eq} = \frac{D_2^2 - D_1^2}{D_1}}$$

SIZE REDUCTION OF SOLID'S

Breaking of coarse particles into fine particle.

* Objectives of size Reduction :-

→ to increase surface area
→ Better handling
→ to produce solid particles of desired particles

* Methods of size Reduction :-

There are four basic ways to reduce the size of a material

1) IMPACT (Gravity and Dynamic impact)
2) COMPRESSION
3) ATTRITION
4) SHEAR

(I) IMPACT :- (Gravity & dynamic impact)

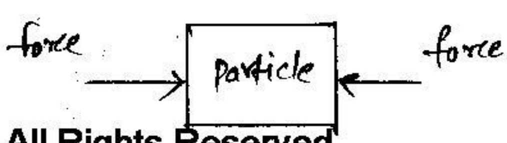
→ sharp and sudden force for an instantaneous time

(a) Dynamic impact :- when both objects are moving
Ex:- cricket bat & ball

(b) Gravity impact :- when one object is moving and other is at rest
Ex:- Coal dropped onto a hard steel surface

(II) COMPRESSION :-

→ When two continous forces applied on a object in Normal direction

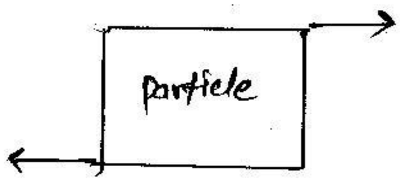


(III) ATTRITION :-

It is a method of size reduction by rubbing or scurbbing the materials between two hard surfaces.

(IV) SHEAR :-

Shear consists of cleaving action rather than the rubbing action associated with attrition.



When two continous force applied to an object in shear direction.

Power consumption in size reduction :-

→ When external stress/force is applied for size reduction, the solid particles at first are twisted and strained.

→ The work required to strain them is stored temporarily in the solids as the mechanical energy of stress.

→ When additional force is applied to these already stressed particles, they are distorted beyond their ultimate strength and are suddenly broken into smaller particles, which ultimately generate new surfaces.

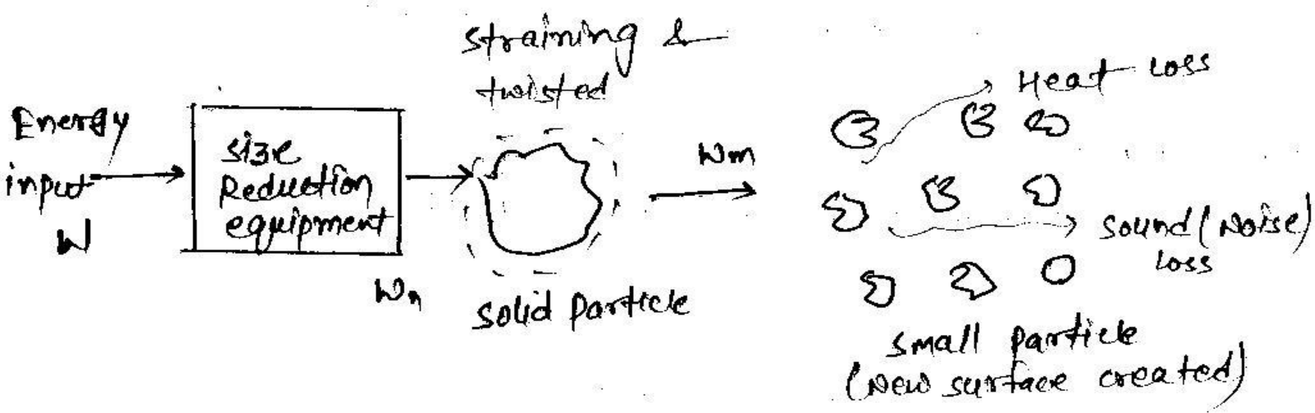


Diagram for creation of a new surface *

Law's of Crushing :-

1) Rittinger's law (feed size < 0.05 mm)
2) Kick's Law (size > 50 mm)
3) Bond's Law (0.05 mm < size < 50 mm)

(1) Rittinger's Law :- (0.05 mm)

Work required for size reduction is directly proportional to new surface area created

$$W_R \propto (A_{ssb} - A_{ssa})$$

$$W_R = K_{es} (A_{ssb} - A_{ssa})$$

unit: $A_{ssb} \rightarrow J/m^2$, $W \rightarrow J/kg$

Specific surface Area $A_w = \frac{6}{\phi_s \rho_p} \sum_{i=1}^{n} \frac{x_i}{\bar{D}_{pi}}$

$$\left(A_{ssb} = \frac{6}{\phi_s \rho_p} \frac{1}{\bar{D}_{pb}} \right)$$

$$\left(A_{ssa} = \frac{6}{\phi_s \rho_p} \frac{1}{\bar{D}_{pa}} \right)$$

$$W_R = k_{es} \frac{6}{\phi_s \rho_p} \left[\frac{1}{\bar{D}_{pb}} - \frac{1}{\bar{D}_{pa}} \right]$$

Rittinger constant $\left(K_R = k_{es} \frac{6}{\phi_s \rho_p} \right)$

Rittinger Number $= \frac{1}{K_R} = \frac{\phi_s \rho_p}{6 k_{es}}$

**

$$\boxed{\frac{P}{\dot{m}} = W_R = K_R \left[\frac{1}{\bar{D}_{pb}} - \frac{1}{\bar{D}_{pa}} \right]}$$

$\dot{m} = kg/sec$

* Determination of power consumption :-

crushing efficiency $\eta_c = \dfrac{\text{total surface energy created}}{\text{total energy absorbed by solid}} = \dfrac{W_m}{W_n}$

Mechanical efficiency $\eta_m = \dfrac{\text{total energy absorbed by solid}}{\text{total energy supplied to feed}} = \dfrac{W_n}{W}$

$$\left\{ \eta_c = \frac{W_m}{W_n} = \frac{e_s (A_{ssb} - A_{ssa})}{W_n} \right\} \quad \text{—— (1)}$$

Where e_s = surface energy per unit area ; J/m^2

A_{ssb} = new surface area created per unit mass ; m^2/kg

A_{ssa} = free surface area per unit mass ; m^2/kg

W_n ; J/kg (energy absorbed by a unit mass of solid)

$$\eta_m = \frac{W_n}{W}$$

$W = \dfrac{W_n}{\eta_m}$; put value of W_n from eq^n (1)

$$\left[W = \frac{e_s (A_{ssb} - A_{ssa})}{\eta_c \eta_m} \right] \; ; \; J/kg$$

* power fed to the equipment $P = W \times \dot{m}$

$$P = \left[\frac{e_s (A_{ssb} - A_{ssa})}{\eta_c \eta_m} \right] \times \dot{m} \; ; \; J/sec = watt$$

FHpNb5S8ciVFB0

* **Limitation of Rittenger's Law's :-**

→ Does not account Mechanical losses

* **Assumption :-**

→ Assume particle density (ρ_p) and particle sphericity (ϕ_s) are constant for feed & product.

(2) Kick's Laws :- (> 50 mm)

The work required for crushing is constant for a given mass of material, for a constant reduction ratio irrespective their initial size.

SAMPLE

$$\text{Reduction Ratio} = \frac{\text{Average diameter of feed particle}}{\text{Average diameter of product particle}}$$

$$W_K = \frac{P}{\dot{m}} = K_K \ln\left\{\frac{\bar{d}_{pA}}{\bar{d}_{pB}}\right\}$$

Where K_K = Kick's constant

$\bar{d}_{pA}$ = Avg. dia. of feed particle

$\bar{d}_{pB}$ = Avg. dia. of product particle

FHpNb5S8ciVFB0

(3) Bond's Law :- (0.05 mm – 50 mm)

The work required for crushing is directly proportional to the square root of surface area to volume ratio of product of diameter d_{PB}.

$$W_B \propto \sqrt{\left(\frac{S_p}{V_p}\right)_b}$$

$$\phi_s = \frac{6/D_p}{S_p/V_p} \Rightarrow \left(\frac{S_p}{V_p}\right) = \frac{6}{D_p\,\phi_s}$$

$$W_R \propto \sqrt{\frac{6}{D_p\,\phi_s}} \Rightarrow W_R = K\sqrt{\frac{6}{D_p\,\phi_s}}$$

$$W_R = \sqrt{\frac{6K^2}{\phi_s}}\cdot\frac{1}{\sqrt{d_{P_b}}}$$

SAMPLE

$$\boxed{W_R = K_B \cdot \frac{1}{\sqrt{d_{P_b}}}}$$

Where

K_B = Bonds constant

$$K_B = \sqrt{\frac{6K^2}{\phi_s}}$$

→ The Bonds Law is also known as "universal Law of crushing"

* **Work INDEX** :- (Gross energy required in kwh/tonn)

"For a crushing of a large size feed up to a size (product) such that 80% of the product can pass through 100 μm mesh screen".

$$\text{work index} = w_i = K_b\,\frac{1}{\sqrt{0.1\,mm}}$$

unit $w_i \rightarrow$ kwh/tonn ; $\boxed{K_b = 0.3162\,w_i}$

***

$$W_B = \frac{P}{\dot{m}} = 0.3162\, W_i \left\{ \frac{1}{\sqrt{d_{PB}}} - \frac{1}{\sqrt{d_{PA}}} \right\}$$

FHpNb5S8ciVFB0

Imp.r

① Rittenger's Law :- (< 0.05 mm)

$$\left[W_R = \frac{P}{\dot{m}} = K_R \left\{ \frac{1}{d_{PB}} - \frac{1}{d_{PA}} \right\} \right]$$

P → kW, $\dot{m}$ → tonn/hr, d_{PB} → m, d_{PA} → m

② Kick's law (> 50 mm)

$$\left[W_R = \frac{P}{\dot{m}} = K_K \ln \left\{ \frac{d_{PA}}{d_{PB}} \right\} \right]$$

P → kW, $\dot{m}$ → tonn/hr, $\frac{d_{PA}}{d_{PB}}$ → for any unit

SAMPLE

③ Bond's Law : (0.05 mm – 50 mm)

$$\left[W_B = \frac{P}{\dot{m}} = 0.3162\, W_i \left\{ \frac{1}{\sqrt{d_{PB}}} - \frac{1}{\sqrt{d_{PA}}} \right\} \right]$$

P → kW, $\dot{m}$ → tonn/hr, d_{PB} → mm, d_{PA} → mm

* Generalised law :-

$$\left[d(W) = d\left(\frac{P}{\dot{m}}\right) = -K \frac{d(\bar{D}_{vs})}{(\bar{D}_{vs})^n} \right]$$

get Rittinger's law $n = 2$

Kick's law $n = 1$

Bond's law $n = 1.5$

Que→ 8) Particles of average feed size of 50×10^{-4} m are crushed to an average product size of 10×10^{-4} m at the rate of 20 tonnes per hour. At this rate crusher consumes 40 kW of power of which 5 kW are required for running the mill empty. Calculate the power consumption if 12 tonnes/hr of this product is further crushed to 5×10^{-4} m size in the same mill. Assume that Rittinger's law is applicable.

Sol→

$$\left\{ W = \frac{P}{\dot m} = K_R \left[\frac{1}{d_{PB}} - \frac{1}{d_{PA}} \right] \right\}$$

Actual power consumption P = 40 − 5 = 35 kW

$$\frac{\dfrac{35}{20} = K_R\left[\dfrac{1}{10\times10^{-4}} - \dfrac{1}{50\times10^{-4}}\right]}{\dfrac{P}{12} = K_R\left[\dfrac{1}{5\times10^{-4}} - \dfrac{1}{10\times10^{-4}}\right]}$$

$$\Rightarrow \frac{35}{20}\times\frac{12}{P} = \frac{\left(\frac{1}{10} - \frac{1}{50}\right)}{\left(\frac{1}{5} - \frac{1}{10}\right)} \Rightarrow \boxed{P = 26.25 \text{ kW}}$$ Ans

Que→ 9) A sample of material is crushed in a jaw crusher such that the average size of particles is reduce from 50 mm to 10 mm with the energy consumption of 13 kW/(kg/s). Determine the consumption of energy to crush the same material of 75 mm average size of 25 mm using Rittinger's and Kick's laws.

Sol^n (I) Rittinger's law:

$$\left[\frac{P}{\dot{m}} = K_R\left(\frac{1}{d_{Pa}} - \frac{1}{d_{Fa}}\right)\right]$$

$$\frac{13\ kW}{1\ kg/s} = \frac{13\ kW}{1\times10^{-3}\ tonn/hr \times 3600} = K_R\left(\frac{1}{10} - \frac{1}{50}\right)$$

$$\frac{P}{\dot{m}} = K_R\left(\frac{1}{25} - \frac{1}{75}\right)$$

$$\frac{P}{\dot{m}} = 1.203\ kW\,hr/tonn \quad \textbf{Answer}$$

(II) Kick's Law:

$$\left[\frac{P}{\dot{m}} = K_K \ln\left(\frac{d_{Fa}}{d_{Pa}}\right)\right]$$

SAMPLE

$$\frac{13\ kW}{1\ kg/sec} = \frac{13}{10^{-3}\times3600\ tonn/hr} = K_K \ln\left(\frac{50}{10}\right)$$

$$\frac{P}{\dot{m}} = K_K \ln\left(\frac{75}{25}\right)$$

$$\frac{P}{\dot{m}} = 2.46\ kW\,hr/tonn \quad \textbf{Answer}$$

Ques 10) 270 kW of power is required to crush 150 tonnes/hr of a material. If 80% of the feed passes through a 50-mm screen and 80% of the product passes through a 3-mm screen, calculate the work index of the material. And what will be the power required for the same feed at 150 tonn/hr to be crushed to a product such that 80% is to pass through a 1.5 mm screen?

 | www.chemgateacademy.com