CHEMICAL ENGINEERING (GATE & PSUs)

Postal Correspondence

STUDY MATERIAL (Handwritten Notes)

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HEAT TRASFER

GATE-2022 Syllabus: Chemical Engineering

Equation of energy, Steady and unsteady heat conduction, convection and radiation, thermal boundary layer and heat transfer coefficients, boiling, condensation and evaporation; types of heat exchangers and evaporators and their process calculations. Design of double pipe, shell and tube heat exchangers, and single and multiple effect evaporators.

HEAT TRANSFER COURSE CONTENT

1. **Introduction**
2. **Conduction**
3. **Convection**
4. **Radiation**
5. **Heat Exchanger**
6. **Boiling**
7. **Condensation**
8. **Evaporation**

Note for Student:

1. Full GATE Syllabus covers in Notes.

2. Total number of pages in HT Notes = 250 Pages

3. No. of Questions solved in Notes = 90+ Questions

(GATE PYQs & other good quality question)

HEAT TRANSFER

* **HEAT**:- The heat is form of energy in transit with in the system or from one system to other system due to different in temperature.

* Differente between thermodynamic and Heat transfer :-

1) Thermodynamics :- Thermodynamics able to find the amount of heat transfer b/w two equilibrium state but the thermo-dynamics get fail to find what time required to reach from initial state to final state.

Thermodynamics is an equilibrium phenomena which tells why the heat is going to heat transfer.

2) Heat transfer :- How the heat is going to transfer it is given by steady of heat transfer. Heat transfer may be used to predict the amount energy transfer that may take place between material bodies as a result of a temperature difference.

→ In the desiging of equipment the rate of heat transfer is important rather than amount of heat transfer.

→ Heat transfer is non equilibrium phenomena, only come in picture when we talk about the rate of Heat transfer.

Different modes of heat transfer :-

1) Conduction
2) Convection
3) Radiation

1) <u>CONDUCTION</u> :- When a temperature gradient exists in a body, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region. We say that the energy is transferred by conduction and that heat transfer rate per unit area is proportional to the normal temperature gradient

Heat flux ∝ temp. gradient

$$\frac{Q}{A} \propto \frac{dT}{dx}$$

Heat flux $\quad q_x = \frac{Q}{A_c}$

Fourier's Law of Heat conduction

$$Q = -kA\frac{dT}{dx}$$

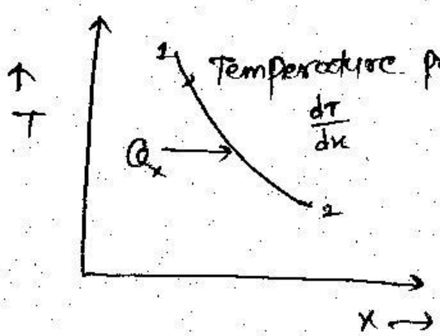
Where Q = Heat transfer rate

$\frac{dT}{dx}$ = temperature gradient in the direction of the heat flow

k = thermal conductivity of the material,

A = Area of heat transfer normal to direction of heat flow.

<u>Note</u>:- minus sign is inserted so that the second principle of thermodynamics will be satisfied; Heat must flow downhill on the temperature scale



* **Thermal conductivity** :- for Isotropic material k is not a function of position. k is only a function of temperature

$$k = f(T)$$

k is called thermal conductivity and it is transport property of material to conduct the heat and this transport property stabilies relationship b/w the flux and gradient

$$\boxed{\vec{q}_x = -k \frac{dT}{dx}} \quad \text{(Heat transfer)}$$

k = thermal conductivity

$$\boxed{N_A = -D_{AB} \frac{dC_A}{dz}} \quad \text{(Mass transfer)}$$

D_{AB} = Diffusivity

$$\boxed{\tau_x = -\mu \frac{dx}{dy}} \quad \text{(fluid mechanics)}$$

μ = viscosity

[flux = transport property * gradient]

k is ability of the material to the heat transfer

$$q_x = +k \frac{dT}{dx}$$

$$k = \frac{q_x}{dT/dx}$$

$$k = \frac{Q/A}{dT/dx}$$

→ K is a material property.

* **unit of K :** $K = \frac{Q/A}{dT/dx} = \frac{W/m^2}{°C/m} = \frac{W}{m°C}$

$\frac{W}{m.K}$ or $\frac{W}{m°C}$

Effect of temp. on thermal conductivity :-

I) **In solid :-** $\boxed{T\uparrow \quad K\downarrow}$

(metal)

→ since as temperature increased in the case of metal due to extra vibration the moment of the electron get decreases and hence thermal conductivity decreases with increases temperature.

SAMPLE

Note: In metal exception . $T\uparrow$, $K\uparrow$ is Aluminum

(Non metal)

→ since in the case of non metals, the maximum portion of heat conducted because of lattic vibration as temp increases the molecules vibrate more frequently so k increases with increasing temp

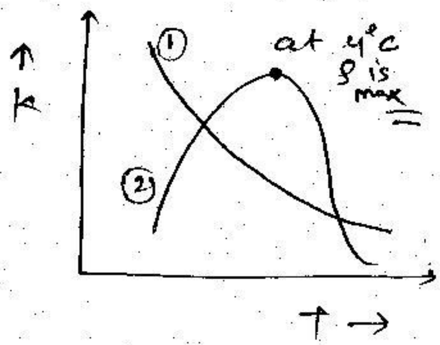
$\boxed{K\uparrow \quad T\uparrow}$

II) **In Gases :-** $\boxed{T\uparrow \quad K\uparrow}$

In the case of gase as temperature increases the velocity of molecules increases and hence number of collision increases and k increases with increases temperature.

→ because increase in moment of gas molecules & increase kinetic energy of molecules

III) for Liquid :- Experimentally shown that the thermal conductivity get decreases with increases temp but exception is water $\boxed{T\uparrow \; k\downarrow}$



① Any liquid
② Water (exception)

* Water : Initially k is increases then k is decrease when increases temperature.

IV) Thermal conductivity of alloy :-

When the two metals are going to add to form an alloy for examples Bronze, Bronze is alloy of copper and Alluminum,

→ In alloy th decrease the flow of free electron hence the thermal conductivity significantly decreases.

$k_{Cu} = 401$ W/mk $\quad k_{Al} = 237 \quad$ Alloy $= 52$ W/mk

$$\left[k_{\text{pure metal}} > k_{\text{metallic Alloy}} > k_{\text{non-metals}} > k_{\text{Liquid}} > k_{\text{insulator}} > k_{\text{gases}} \right]$$

$$\left[k_{\text{copper}} > k_{\text{aluminium}} > k_{\text{low carbon steels}} > k_{\text{Alloy steel}} > k_{\text{brick}} > k_{\text{Ice}} > k_{\text{water}} > k_{\text{gas}} \right]$$

for non Black body :-

$$Q = \varepsilon \sigma A (T_1^4 - T_2^4)$$

ε = Emisivity

$0 < \varepsilon < 1$

CONDUCTION :→

* fourier's law of Heat conduction

$$Q = -KA \frac{dT}{dx}$$

Q = Rate of heat transfer
A = Area perpendicular to the heat flow

* General Heat conduction equation :-

Consider one dimensional steady state heat conduction

→ for Rectangular coordinate

$T = f(x, y, z, t)$; t = time

→ for one dimension

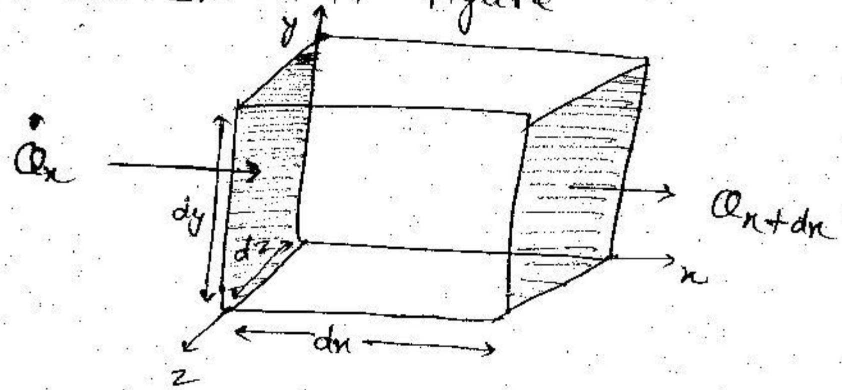
$T = f(x, t)$

* Assumptions :-

ⓐ steady state conduction

ⓑ one-dimensional heat flow

Consider a thin element of thickness Δx in a large plane wall as shown in figure



applying heat balance on elemental thickness.

$$\begin{bmatrix}\text{Rate of Heat}\\ \text{flow in}\end{bmatrix} - \begin{bmatrix}\text{Rate of Heat}\\ \text{flow out}\end{bmatrix} + \begin{bmatrix}\text{Generation of}\\ \text{Heat within solid}\end{bmatrix} = \begin{bmatrix}\text{Accumulation}\end{bmatrix}$$

$$\dot{Q}_x - \dot{Q}_{x+dx} + \dot{q}\, dx\, dy\, dz = \frac{\partial}{\partial t}(\rho \Delta V C_p T)$$

unit J/s = Watt Watt $\frac{W}{s} \cdot \frac{W}{m^3} \times m^3$ $\frac{1}{s}\left(\frac{kg}{m^3} \times m^3 \times \frac{J}{kg \cdot K} \times K\right)$

$\frac{J}{s}$ = Watt

volume $\Delta V = dx dy dz$

$$\dot{Q}_x - \dot{Q}_{x+dx} + \dot{q}\, dx dy dz = \rho\, dx dy dz\, C_p \frac{dT}{dt}$$

dividing equation by $dx dy dz$ we get

$$\left(\frac{\dot{Q}_x - \dot{Q}_{x+dx}}{dx}\right)\frac{1}{dy dz} + \dot{q} = \rho C_p \frac{dT}{dt}$$

taking limit as $dx \to 0$

$$-\frac{d}{dx}(\dot{Q}_x)\frac{1}{dy\, dz} + \dot{q} = \rho C_p \frac{dT}{dt}$$

Now from fourier's law $\qquad A = dz\,dy$

$$\dot{Q}_x = -k\,dz\,dy\,\frac{dT}{dx} \qquad \text{——— (1)}$$

$$-\frac{d}{dx}(\dot{Q}_x)\frac{1}{dy\,dz} + \dot{q} = \rho C_p \frac{dT}{dt} \qquad \text{——— (2)}$$

$$-\frac{d}{dx}\left\{-k\,dy\,dz\,\frac{dT}{dx}\right\}\frac{1}{dy\,dz} + \dot{q} = \rho C_p \frac{dT}{dt}$$

$$k\frac{d^2T}{dx^2} + \dot{q} = \rho C_p \frac{dT}{dt}$$

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = \frac{\rho C_p}{k}\frac{dT}{dt}$$

$$\boxed{\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{dT}{dt}}$$

Where $\left(\alpha = \frac{k}{\rho C_p}\right) \rightarrow$ Thermal diffusivity

In case of three dimensions

Imp.
$$\boxed{\frac{\partial^2T}{\partial x^2} + \frac{\partial^2T}{\partial y^2} + \frac{\partial^2T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

vector form

$$\boxed{\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

$$\nabla = \text{grad} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

ⓐ for one-dimension, no heat generation

$$\frac{d^2T}{dx^2} = \frac{1}{\alpha}\frac{dT}{dt}$$ fourier equation

ⓑ for steady state, no heat generation, one-Dimension

$$\frac{d^2T}{dx^2} = 0$$ Laplace equation

ⓒ steady state, one dimension

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$ Poisson's equation

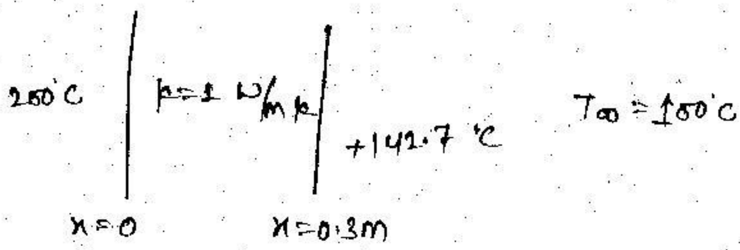
physical significance of (α) → (Thermal diffusivity)

$$\alpha = \frac{k}{\rho C_p} = \frac{\text{Rate of Heat flow}}{\text{storage of Heat}}$$

→ α measures the penetration power of heat or it measures how fast heat is going to transfer from a system. It also measures the relative importance of heat transport through the material to the heat stored in the material.

→ If α is high then large amount of heat will flow through the system.

Ques 1) If the temperature distribution in the slab is given by following equation $T(x) = 200 - 200x + 30x^2$



In this profile Temp. in °C and x in m find out

(I) surface heat ~~transfer~~ flux ?

(II) Rate of change of energy storage per unit area.

(III) find the heat transfer coefficient ?

Sol → (I) $T(x) = 200 - 200x + 30x^2$

$$\boxed{q = -k\frac{dT}{dx}} \qquad \frac{dT}{dx} = -200 + 60x$$

(x = 0) $q_0 = -(1)[-200 + 60(0)] = 200 \ W/m^2$

(x = 0.3) $q_{x=0.3} = -(1)[-200 + 60(0.3)] = 182 \ W/m^2$

(II) Rate of change $Q = q_0 - q_L$

$= 200 - 180 = 18 \ W/m^2$

(III) $$\boxed{q_L = h(T_{out} - T_\infty)}$$

$182 = h(142.7 - 150)$

$h = 4.28 \ \frac{W}{m^2 \cdot K}$ Answer

Ques 2) What is the mean radius which is responsible for heat transfer become r_1 & r_2 in Hollow cylinder.

(A) $\frac{r_1+r_2}{2}$ (B) $\frac{1}{r_1}+\frac{1}{r_2}$ (C) $\sqrt{r_1 r_2}$ (d) ✓ $\frac{r_2-r_1}{\ln r_2/r_1}$

Sol→ $Q = -kA\frac{dT}{dr}$ $A = (2\pi r)\cdot L$

$$Q = -k(2\pi r_m L)\frac{(T_2-T_1)}{(r_2-r_1)}$$

r_m = mean radius

$$Q = \frac{(T_1-T_2)}{\frac{(r_2-r_1)}{k2\pi r_m L}}$$

for Hollow cylinder

$$Q = \frac{(T_1-T_2)}{\frac{1}{k2\pi L}\ln\frac{r_2}{r_1}}$$

SAMPLE

compare each term

$$\frac{r_2-r_1}{k2\pi r_m L} = \frac{1}{k2\pi L}\ln\frac{r_2}{r_1}$$

$$\boxed{r_m = \frac{r_2-r_1}{\ln\frac{r_2}{r_1}}}$$ = Lograthmic mean radius for cylinder

Ques 3) What is mean radius which is responsible for heat transfer become r_1 & r_2 in Hollow sphere :

Sol→ $Q = -k(4\pi r_m^2)\frac{(T_2-T_1)}{(r_2-r_1)}$ $Q = \frac{(T_1-T_2)}{\frac{1}{k4\pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$

$$Q = \frac{(T_1-T_2)}{\left(\frac{r_2-r_1}{k4\pi r_m^2}\right)}$$ compare each term

$$\frac{r_2-r_1}{k4\pi r_m^2} = \frac{1}{k4\pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \Rightarrow \boxed{r_m = \sqrt{r_1 r_2}}$$ = Geometric mean radius

Summary :-

for steady state, one-D, no internal heat generation

Coordinate system	Rate of Heat transfer	Resistance
Rectangular	$Q = \frac{(T_1 - T_2)}{(L/kA)}$	$\frac{L}{kA}$
cylindrical	$Q = \frac{(T_1 - T_2)}{\frac{1}{k\,2\pi L} \ln r_2/r_1}$	$\frac{\ln r_2/r_1}{k\,2\pi L}$
spherical	$Q = \frac{(T_1 - T_2)}{\frac{1}{k\,4\pi}\left(\frac{r_2 - r_1}{r_1 r_2}\right)}$	$\frac{r_2 - r_1}{k\,4\pi\, r_1 r_2}$

SAMPLE

<GATE 2005>

Ques 4) A copper tube of outer diameter 5 cm & inner diameter 4 cm is used to convey hot fluid. The inner surface of the tube at temperature of 80°C while the outer surface is at temperature of 25°C, what is the rate of heat transfer across the tube per meter length of steady state k of tube 10 W/m·K.

Soln

cylinder

$$Q = \frac{(T_1 - T_2)}{\frac{1}{k\,2\pi L} \ln \frac{r_2}{r_1}} \Rightarrow \frac{Q}{L} = \frac{(80-25)}{\frac{1}{10 \times 2 \times 3.14} \times \ln(5/4)}$$

$$\frac{Q}{L} = 15487 \text{ W/m}$$

Answer

(GATE-2006)

Ques 5) A stagnant liquid film 0.4 mm thickness is held b/w two parallel plate the top plate is maintained at 40°C & bottom plate is maintained at 30°C. The k of liquid 0.14 W/mK. Then the steady state heat flux. Assuming one dimensional heat transfer is

(A) 3.5 (B) 350 (C) 3500 ✓ (D) 7000

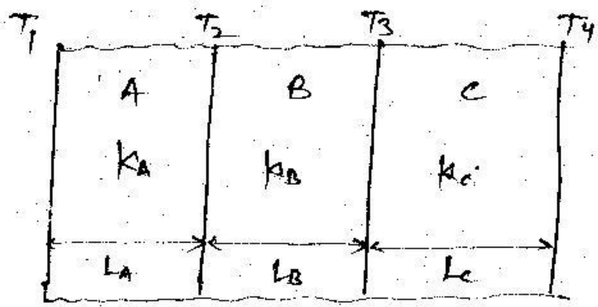
Sol → Comp plan wall

$$Q = \frac{T_1 - T_2}{\Delta x / kA}$$

$$\frac{q}{A} = \frac{(40-30)}{\frac{(0.4 \times 10^{-3})}{0.14}} \quad ; \quad \Delta x = 0.4 \text{ mm}, \ \Delta x = 0.4 \times 10^{-3} \text{ m}$$

$$\frac{q}{A} = 3500 \ W/m^2 \quad \text{Answer}$$

Heat transfer in parallel composite plate :- (series)



Heat transfer by conduction

$$\frac{Q}{A} = \frac{\Delta T}{L/kA}$$

$$\frac{Q}{A} = \frac{T_1 - T_2}{L_A/K_A} = \frac{T_2 - T_3}{L_B/k_B} = \frac{T_3 - T_4}{L_C/k_C}$$

$$T_1 - T_2 = \frac{Q}{A} \frac{L_A}{K_A} \quad \text{—— (1)}$$

$$T_2 - T_3 = \frac{Q}{A} \cdot \frac{L_B}{K_B} \quad \text{—— (2)}$$

$$T_3 - T_4 = \frac{Q}{A} \cdot \frac{L_C}{k_C} \quad \text{—— (3)}$$

add equation ①, ② & ③

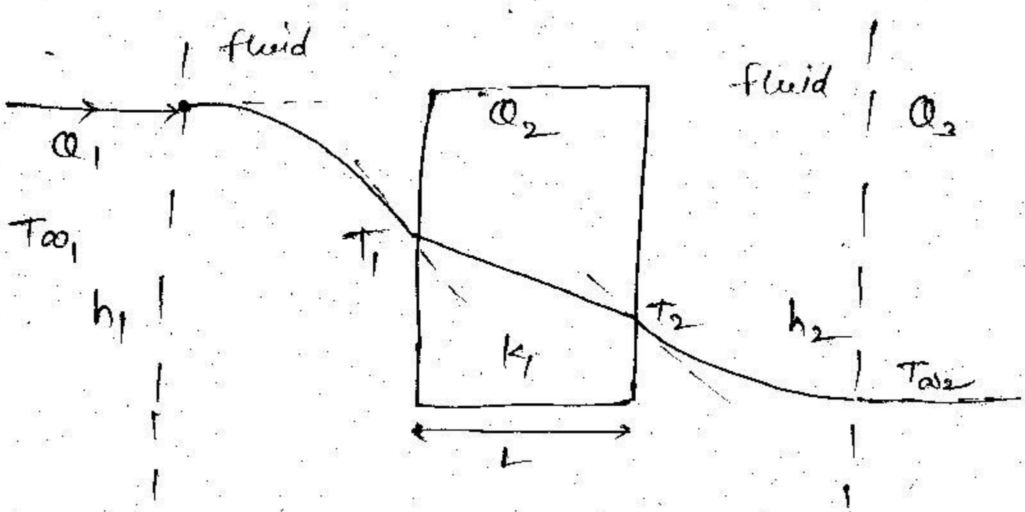
$$T_1 - T_4 = \frac{Q}{A} \cdot \frac{L_A}{k_A} + \frac{Q}{A}\frac{L_B}{k_B} + \frac{Q}{A}\frac{L_C}{k_C}$$

$$T_1 - T_4 = \frac{Q}{A}\left[\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}\right]$$

so
$$\boxed{\frac{Q}{A} = \frac{(T_1 - T_4)}{\left[\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}\right]}}$$

Resistence $\left(R_{total} = \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}\right)$

Combined form of Heat Transfer :- (plane wall)



$$Q_1 = h_1 A_1 (T_{\infty_1} - T_1) \quad \text{—— ①}$$ convection (fluid to solid)

$$Q_2 = \frac{T_1 - T_2}{L/k_1 A} \quad \text{—— ②}$$ conduction (solid to solid)

$$Q_3 = h_2 A_1 (T_2 - T_{\infty_2}) \quad \text{—— ③}$$ convection (solid to fluid)

Now

$$T_{\infty_1} - T_1 = \frac{Q_1}{h_1 A_1} \quad \text{--- (4)}$$

$$T_1 - T_2 = \frac{Q_2 L}{k_1 A} \quad \text{--- (5)}$$

$$T_2 - T_{\infty_2} = \frac{Q_3}{h_2 A_1} \quad \text{--- (6)}$$

Now adding eq (4), (5) and (6)

$$T_{\infty_1} - T_{\infty_2} = \frac{Q_1}{h_1 A_1} + \frac{Q_2}{k_1 A} + \frac{Q_2}{h_2 A_1}$$

Now at steady state

$Q_1 = Q_2 = Q_3$ (Heat transfer rate)

SAMPLE

$$T_{\infty_1} - T_{\infty_2} = Q\left\{\frac{1}{h_1 A_1} + \frac{L}{k_1 A} + \frac{1}{h_2 A_1}\right\}$$

so

$$\boxed{Q = \frac{T_{\infty_1} - T_{\infty_2}}{\frac{1}{h_1 A_1} + \frac{L}{k_1 A} + \frac{1}{h_2 A_1}}}$$

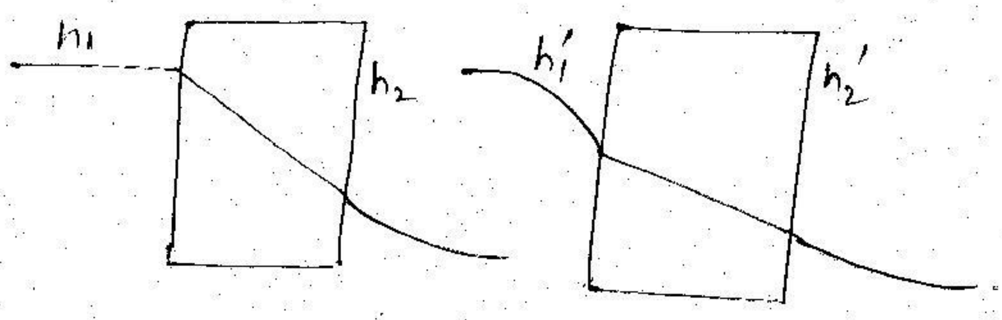
or

$$\left(Q = \frac{T_{\infty_1} - T_{\infty_2}}{R_1 + R_2 + R_3}\right)$$

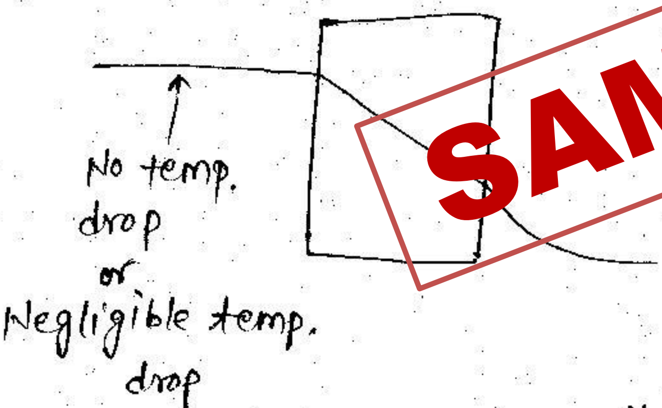
Where thermal Resistence

$$R_1 = \frac{1}{h_1 A_1}; \quad R_2 = \frac{L}{k_1 A}, \quad R_3 = \frac{1}{h_2 A_1}$$

Note :-



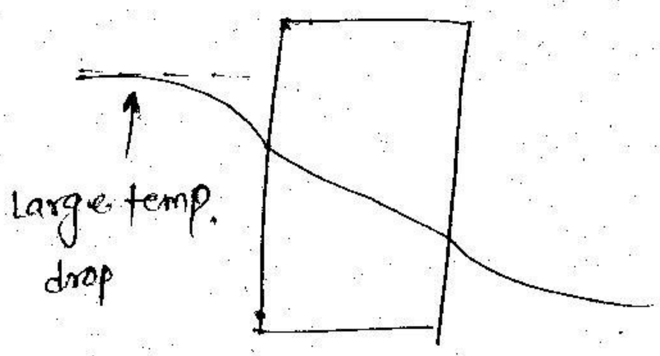
from the following figure two profile of temp given in plane wall. In both same rate of Heat transfer is there then what is the relation between h_1 and h_1'



so $Q = h_1 A_1 (\Delta T) \rightarrow$ case A

$Q = h_1' A_1 (\Delta T) \rightarrow$ case B

$(\Delta T)_A < (\Delta T)_B$



* so in order to keep the amount of heat Q same in both cases h_1 in case (A) Should be greater than h_1' in case (B)

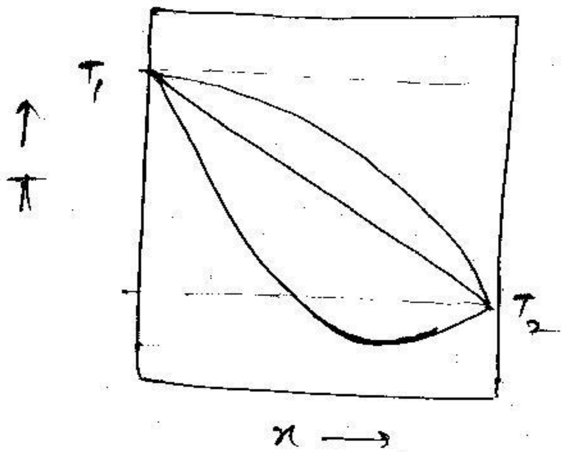
so $\boxed{h_1 > h_1'}$

if h is maximum then Q is maximum

$$Q = hA(\Delta T)$$

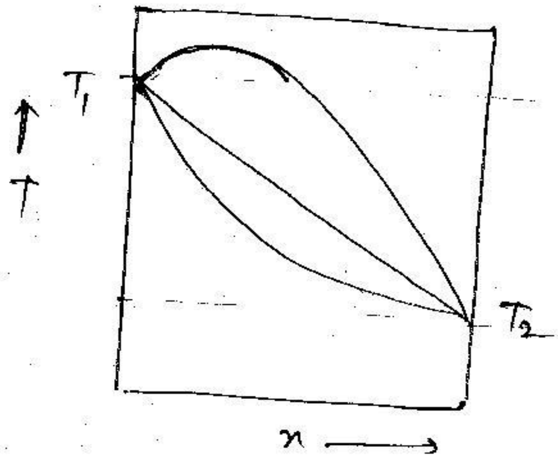
$$Q \propto h$$

Case → ①



This type of profile only possible when internal Heat consumption is there

Case → ②

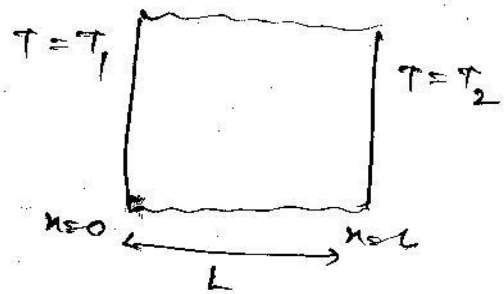


This type of profile is possible when internal heat generation is there in the system

SAMPLE

Develop the expression for Heat transfer in plan wall :-

(K varies with temp.)



if $K = K_0(1+\alpha T)$

H.T rate $\boxed{q = -KA\frac{dT}{dx}}$

$$q = -K_0(1+\alpha T)A\frac{dT}{dx}$$

$$\int_0^L dx = -\int_{T_1}^{T_2} K_0(1+\alpha T)\frac{A}{q}\,dT$$

$$L = -\frac{K_0 A}{q}\int_{T_1}^{T_2}(1+\alpha T)\,dT$$

$$L = -\frac{K_0 A}{q}\left[T + \frac{\alpha T^2}{2}\right]_{T_1}^{T_2}$$

$$q = \frac{-k_0 A}{L}(T_2 - T_1)\left[1 + \frac{\alpha}{2}(T_2 + T_1)\right]$$

$$Q = \frac{-k_0 A}{L}(T_2 - T_1)\left[1 + \alpha\left(\frac{T_1 + T_2}{2}\right)\right]$$

$\left(\frac{T_1+T_2}{2}\right) \rightarrow T_m$

$$Q = \frac{-A(T_1 - T_2)}{L} k_0 \left[1 + \alpha T_m\right]$$

$$Q = (T_1 - T_2)\frac{A k_m}{L} \qquad \Big| \quad k_m = k_0(1 + \alpha T)$$

$$\boxed{Q = \frac{T_1 - T_2}{L/k_m A}}$$

[illegible] $= \frac{k_1 + k_2}{2}$

↳ Heat conduction expression for plane wall with k varies with temp.

Fins ⟨ Extended surface ⟩ :-

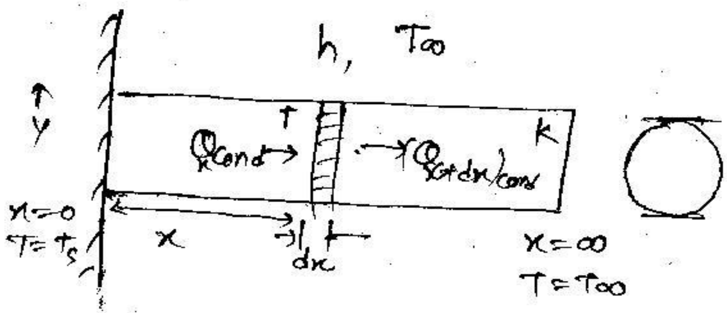
Fins are generally used to increase the rate of heat transfer. fins are projection which are established on a heat surface and they are wear for increase the heat transfer rate by increasing the area of heat transfer.

for eg: condenser tube, refrigerator, in electrical applicances.

1) Fins of Infinite Length

2) fin of finite length and other tip of the fin is insulated.

(1) Fin of Infinite Length ⇒



$$(Q_x)_{cond} = (Q_{dx})_{conv} + (Q_{x+dx})_{conduc}$$

$$\Rightarrow \quad -kA\frac{dT}{dx} = hA'\Delta T + \left[(Q_x)_{cond} + \frac{d}{dx}(Q_x)_{cond}\,dx\right]$$

$A = \pi r^2$

Perimeter $= P\,dx$

$= (2\pi r)\,dx$

$$\Rightarrow \quad -kA\frac{dT}{dx} = hA'\Delta T + \left[-kA\frac{dT}{dx} + \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)dx\right]$$

$$kA\frac{d^2T}{dx^2}dx = hP\,dx\,(T - T_\infty)$$

$$\boxed{\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0} \quad \text{—— (1)}$$

Let $T - T_\infty = y$

$\frac{dT}{dx} = \frac{dy}{dx}$

& $\frac{d^2T}{dx^2} = \frac{d^2y}{dx^2}$

Let $\frac{hP}{kA} = m^2$

$$\boxed{m = \sqrt{\frac{hP}{kA}}}$$

$$\boxed{\frac{d^2y}{dx^2} - m^2 y = 0} \quad \text{—— (2)}$$

$$(D^2 - m^2)\, y = 0$$

solution of this type of differential equation is given by

$$y = c_1 e^{mx} + c_2 e^{-mx}$$

$$\left[(T - T_\infty) = c_1 e^{mx} + c_2 e^{-mx} \right] \quad \text{——— (3)}$$

* Apply Boundary condition

(i) $x = 0 \quad T = T_s$

$$T - T_\infty = c_1 + c_2 \quad \text{——— (4)}$$

(ii) $x = \infty, \quad T = T_\infty$

$$0 = c_1 e^{\infty} + c_2 e^{-\infty}$$

$$0 = c_1 e^{\infty} \qquad (e^{\infty} = \infty)$$

so c_1 must be zero

$$\boxed{c_1 = 0} \quad \text{——— (5)}$$

from eq^n (4)

$$T_s - T_\infty = 0 + c_2$$

$$c_2 = T_s - T_\infty \quad \text{——— (6)}$$

Now we get $(T - T_\infty) = 0 + (T_s - T_\infty)\, e^{-mx}$

$$T - T_\infty = (T_s - T_\infty)\, e^{-mx}$$

$$\left[\frac{T - T_\infty}{T_s - T_\infty} = e^{-mx} \right] \quad \text{——— (7)}$$

$$\boxed{\frac{T - T_\infty}{T_s - T_\infty} = e^{-mx}} \quad \text{--- (7)}$$

$$m = \sqrt{\frac{hP}{kA}}$$

$$\frac{dT}{dx} = (T_s - T_\infty)\, e^{-mx} (-m)$$

$$(Q_{Fin})_{cond} = -kA \left.\frac{dT}{dx}\right|_{x=0}$$

$$= -kA \left[(T_s - T_\infty)\, e^{-mx} (-m) \right]_{x=0}$$

$$= mkA (T_s - T_\infty)$$

$$= \sqrt{\frac{hP}{kA}} \cdot kA \, (T_s - T_\infty)$$

Imp.

$$\boxed{(Q_{Fin})_{cond} = \sqrt{hPkA_c}\,(T_s - T_\infty)} \quad \text{--- (8)}$$

* Proove $(Q_{Fin})_{conduction} = (Q_{Fin})_{convection} \Rightarrow Q = hA\Delta T$

$$(Q_{Fin})_{conv} = \int_0^\infty hP\,dx\,(T - T_\infty)$$

put $(T - T_\infty) = (T_s - T_\infty)\, e^{-mx}$

$$= \int_0^\infty hP\,(T_s - T_\infty)\, e^{-mx}\, dx$$

$$= hP(T_s - T_\infty) \int_0^\infty e^{-mx}\, dx$$

$$= hP(T_s - T_\infty) \left.\frac{e^{-mx}}{-m}\right|_0^\infty$$

$$= hP(T_s - T_\infty)\left(-\frac{1}{m}\right)\left[e^{-\infty} - e^{0}\right]$$

$$(Q_{fin})_{conv} = \frac{hP}{m}(T_s - T_\infty)$$

$$= \frac{hP\,(T_s - T_\infty)}{\sqrt{hP/kA}}$$

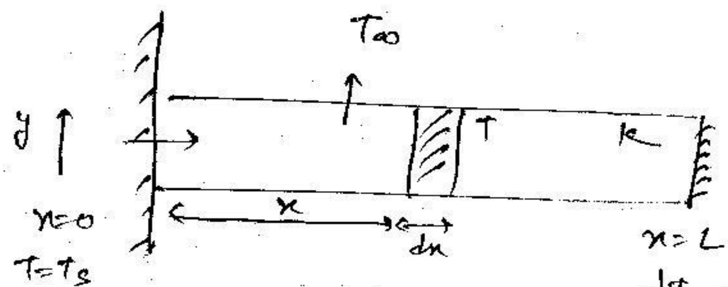
$$(Q_{fin})_{conv} = \sqrt{hPkA}\,(T_s - T_\infty) \quad \text{——— (9)}$$

from eq^n (8) & eq^n (9)

$$(Q_{fin})_{conduction} = (Q_{fin})_{convection}$$

→ Whatever Heat coming from the base of the fin by conduction is equal to the heat reject by convection from the portion of the fin.

⟨2⟩ Fin of finite length and other tip of the fin is Insulated :-



$$\frac{dT}{dx} = 0 \quad \text{means} \quad -kA\frac{dT}{dx} = 0$$

from eq^n (3)
$$\boxed{T - T_\infty = c_1 e^{mx} + c_2 e^{-mx}}$$

$$\because \quad \frac{e^x + e^{-x}}{2} = \cosh x \quad \Big| \quad \frac{e^x - e^{-x}}{2} = \sinh x$$

Adding $\quad e^x = \cosh x + \sinh x$

Subtracting $\quad e^{-x} = \cosh x - \sinh x$

$$T - T_\infty = c_1 e^{mx} + c_2 e^{-mx}$$

$$= c_1(\cosh mx + \sinh mx) + c_2(\cosh mx - \sinh mx)$$

$$T - T_\infty = (c_1 + c_2)\cosh mx + (c_1 - c_2)\sinh mx$$

$$\boxed{T - T_\infty = a \cosh mx + b \sinh mx} \quad \text{——— (10)}$$

↳ General equation for fin

<u>B.C</u>: (i) $x = 0 \quad T = T_s$

$$T_s - T_\infty = a \quad \text{——— (11)}$$

(ii) $x = L, \quad \frac{dT}{dx} = 0$

$$\left.\frac{dT}{dx}\right|_{x=L} = ma \sinh mx + mb \cosh mx \Big|_{x=L}$$

$$0 = m[a \sinh mL + b \cosh mL]$$

$$m \neq 0; \quad a \sinh mL + b \cosh mL = 0$$

$$b = -a \frac{\sinh mL}{\cosh mL}$$

$$b = -(T_s - T_\infty)\frac{\sinh mL}{\cosh mL} \quad \text{——— (12)}$$

from eq^n (10)

$$T - T_\infty = (T_s - T_\infty)\cosh mx - (T_s - T_\infty)\frac{\sinh mL}{\cosh mL} \cdot \sinh mx$$

$$\frac{T - T_\infty}{T_s - T_\infty} = \cosh mx - \frac{\sinh mL}{\cosh mL} \cdot \sinh mx$$

$$\frac{T - T_\infty}{T_s - T_\infty} = \frac{\cosh mL \cdot \cosh mx - \sinh mL \cdot \sinh mx}{\cosh mL}$$

$$\because \cos(A - B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\boxed{\frac{T-T_\infty}{T_s-T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}} \quad \text{--- (13)}$$

↳ Temperature distribution profile

$$(Q_{fin})_{cond} = -kA \left.\frac{dT}{dx}\right|_{x=0}$$

$$= -kA\,(ma \sinh mx + mb \cosh mx)\Big|_{x=0}$$

$$= -kAmb \quad ; \quad b = -(T_s - T_\infty)\frac{\sinh mL}{\cosh mL}$$

$$= -kAmb$$

$$= -kAm\left[-(T_s-T_\infty)\frac{\sinh mL}{\cosh mL}\right]$$

$$= mkA\,(T_s - T_\infty)\tanh mL$$

$$(Q_{fin})_{cond} = \sqrt{\frac{hP}{kA}}\,kA\,(T_s-T_\infty)\tanh mL$$

$$\boxed{(Q_{fin})_{cond} = \sqrt{hPkA}\,(T_s-T_\infty)\tanh mL} \quad \text{--- (14)}$$

SAMPLE

Fin efficiency :- (η)

efficiency of fin is defined as the ratio of

$$\left\{ \eta = \frac{\text{Actual Heat transfer from the fin}}{\text{Maximum Heat transfer if the entire surface of fin ~~if entire~~ maintained at base temp of fin}} \right\}$$

$$\eta_{fin} = \frac{\sqrt{hPkA}\,(T_s-T_\infty)\tanh mL}{h(PL)(T_s-T_\infty)} = \frac{Q_{actual}}{Q_{max}}$$

$A_s \rightarrow PL$ (surface Area)

where $A_s \rightarrow$ surface area

for Rectangular $= P \times L$

for circular $= \pi d L$

① **efficiency in case (I) :-**

$$\eta_{fin} = \frac{Q_{actual}}{Q_{max}} = \frac{\sqrt{hPkA}\,(T_s - T_\infty)}{hPL\,(T_s - T_\infty)}$$

$$\eta_{fin} = \sqrt{\frac{kA}{hP}} \times \frac{1}{L} = \frac{1}{mL}$$

$$\boxed{\eta_{fin} = \frac{1}{mL}} \qquad m = \sqrt{\frac{hP}{kA}}$$

② **efficiency for case (2) :-**

$$\eta_{fin} = \frac{Q_{actual}}{Q_{max}} = \frac{\sqrt{hPkA}\,(T_s - T_\infty)\tanh mL}{h(PL)\,(T_s - T_\infty)}$$

$$\eta_{fin} = \sqrt{\frac{KA}{hP}} \times \frac{1}{L} \tanh mL$$

$$\boxed{\eta_{fin} = \frac{\tanh mL}{mL}}$$