CHEM GATE ACADEMY

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CHEMICAL ENGINEERING

AS PER GATE-2022

Handwritten Notes by Ajay Sir

Fluid Mechanics

Fluid Mechanics

CHEMICAL : GATE & PSUs

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CHEMICAL ENGINEERING (GATE & PSUs)

Postal Correspondence

STUDY MATERIAL (Handwritten Notes)

By Ajay Sir

FLUID MECHANICS

GATE-2022 Syllabus: Chemical Engineering

Fluid statics, surface tension, Newtonian and non-Newtonian fluids, transport properties, shell-balances including differential form of Bernoulli equation and energy balance, equation of continuity, equation of motion, equation of mechanical energy, Macroscopic friction factors, dimensional analysis and similitude, flow through pipeline systems, velocity profiles, flow meters, pumps and compressors, elementary boundary layer theory, Turbulent flow: fluctuating velocity, universal velocity profile and pressure drop.

FLUID MECHANICS COURSE CONTENT

1. **Basic Fluid Mechanics**
2. **Fluid Statics**
3. **Fluid Kinematics**
4. **Fluid Dynamics**
5. **Laminar and Viscous Flow**
6. **Dimensional Analysis**
7. **Flow Meters**
8. **Pump**
9. **Turbulent Flow**

Note for Student:

1. Full GATE Syllabus covers in Notes.

2. Total number of pages in FM Notes = 245 Pages

3. No. of Questions solved in Notes = 110+ Questions

(GATE PYQs & other good quality question)

Fluid Mechanics

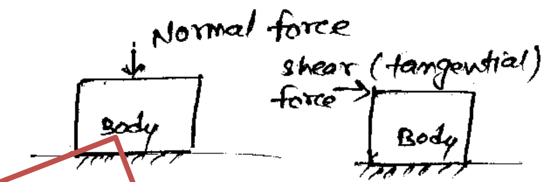
Fluid :-

Fluid is a substance which has ability to flow.
A fluid is a substance that deforms continuously when subjected to a tangential or shear stress, however small the shear stress may be.

* Definition of stress :-

δF_n = Normal force

δF_t = tangential force



consider a small δA on the surface of a body

* fig: forces on a surface

(1) Normal force (δF_n) along the normal to the area δA

(2) tangential force (δF_t) along the plane of δA.

When they are expressed as force per unit area they are called as Normal ~~shear~~ stress and tangential or shear stress

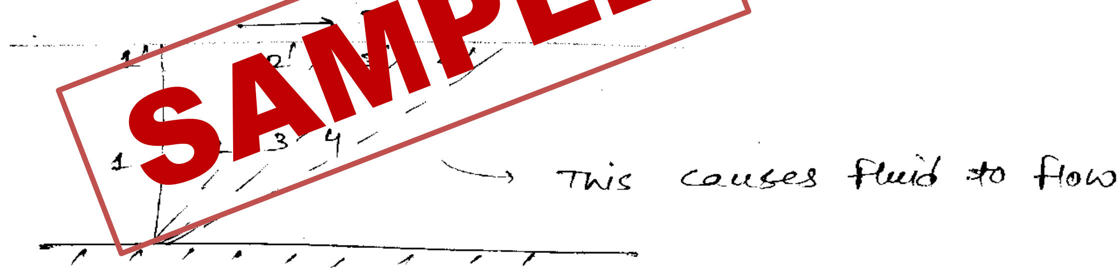
(i) Normal stress $\sigma = \lim_{\delta A \to 0} \frac{\delta F_n}{\delta A}$ $\quad \left(P = \frac{F}{A}\right)$

(ii) Shear stress $\tau = \lim_{\delta A \to 0} \frac{\delta F_t}{\delta A}$ $\quad \left(\tau = \frac{F_s}{A}\right)$

* **Fluid:-** fluid is a substance which has ability to flow.
It differss from solid in such a way, when a tangential force applied on the surface of solid the solid deforms with definite amount and after the removal of strees it may regain it's original shape or it may not or partly regain.

But in case of fluid when tangential force is applied on the surface of fluid element then these is a continuous deformation and after removal of force it can never regain their original shape.

But these is an exception, for a viscoelastic fluid after removal of load fluid may regain their original shape



Shear stress on a fluid Body

→ A fluid is a substance that does not permently resist deformation. During the change in shap Shear stress exist & its magnitude depends upon the viscosity of fluid & rate of sliding.

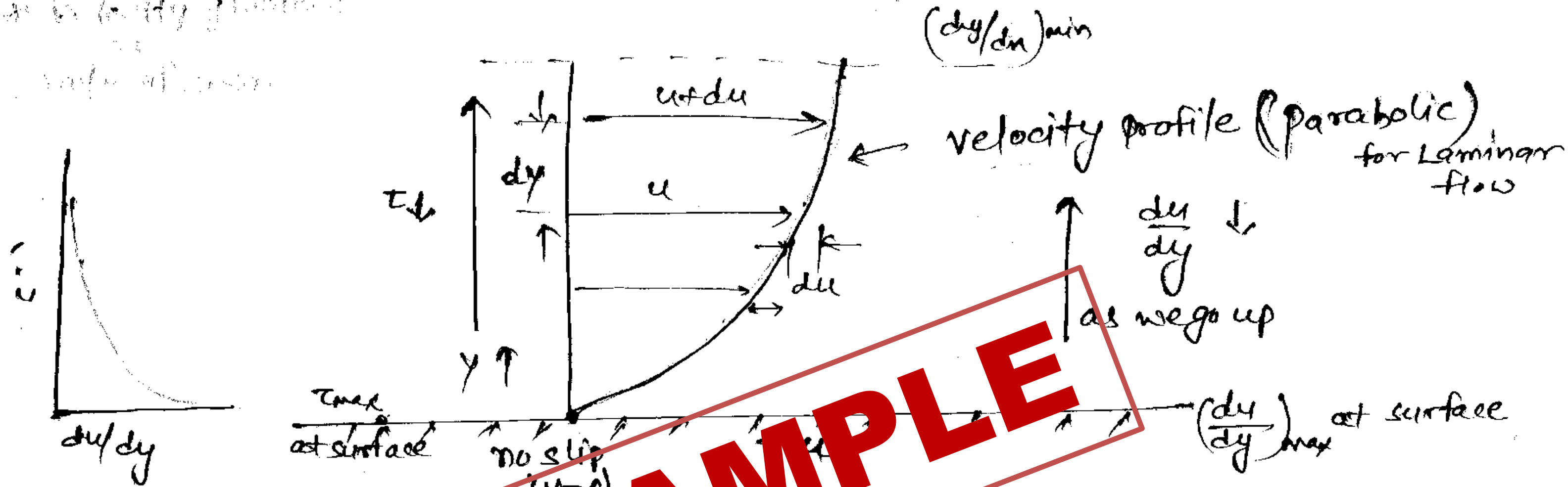
→ Mechanics :- It deals with both stationary and moving bodies under the influence of forces

stationary → Statics
moving bodies → Dynamics

→ Fluid Mechanics : It is defined as science that deals with the behaviour of fluid at rest or in motion and the interaction of fluids with solids or other fluids at the boundaries

Viscosity :- (μ) :- (Dynamic viscosity)

Viscosity is a property of fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.



→ There is no stress when there is no gradient

→ velocity at solid surface is zero (No slip condition)

* The resistance to flow because of internal friction is called viscous resistance and the property which unable the fluid to offer resistance to relative motion is called dynamic viscosity of fluid.

* Newton's Law's of viscosity :-

According to the Newton law's of viscosity the shearing stress is proportional to rate of shear strain (velocity gradient) in the direction normal to flow.

$$\tau \propto \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

where μ = coefficient of dynamic viscosity

$\frac{du}{dy}$ = velocity gradient / rate of shear strain, or rate of shear deformation

$$\left[\mu = \frac{\tau}{du/dy} \right]$$

→ viscosity is also defined as the shear stress required to produce unit rate of shear strain.

* <u>units of viscosity</u> :-

<u>S.I. unit</u> $\mu = \frac{\tau}{du/dy} = \frac{N/m^2}{\frac{m/s}{m}} = \frac{N\text{-}sec}{m^2}$ = Pa-sec

$$\boxed{\mu \rightarrow \text{Pa-sec}} \quad \text{or,} \left(\frac{kg}{m\text{-}sec}\right)$$

<u>C.G.S</u> $\mu \rightarrow 1 \frac{N\text{-}sec}{m^2} = \frac{10^5 \, dyne\text{-}sec}{10^4 \, cm^2} = 10 \frac{dyne\text{-}sec}{cm^2}$

$$\left(1 \text{ poise} \rightarrow 1 \frac{dyne\text{-}sec}{cm^2}\right)$$

$$\left[\mu \rightarrow 10 \text{ poise} = 1 \frac{N\text{-}sec}{m^2}\right]$$

1 poise = 10^{-1} Pa-sec

$\boxed{1 \text{ poise} = 0.1 \text{ Pa-sec}}$; (1 Pa-sec = 10 poise)

1 poise = 100 centipoise

MKS unit $\quad \mu \rightarrow \dfrac{\text{kgf sec}}{m^2}$

CGS unit $\quad \mu \rightarrow \dfrac{\text{dyne-sec}}{cm^2}$

SI unit $\quad \mu \rightarrow \dfrac{\text{N-sec}}{m^2}$ or Pa-sec

$$\left[\frac{1 \text{ kgf sec}}{m^2} = 9.81 \frac{\text{N-sec}}{m^2}\right] = 98.1 \text{ Poise}$$

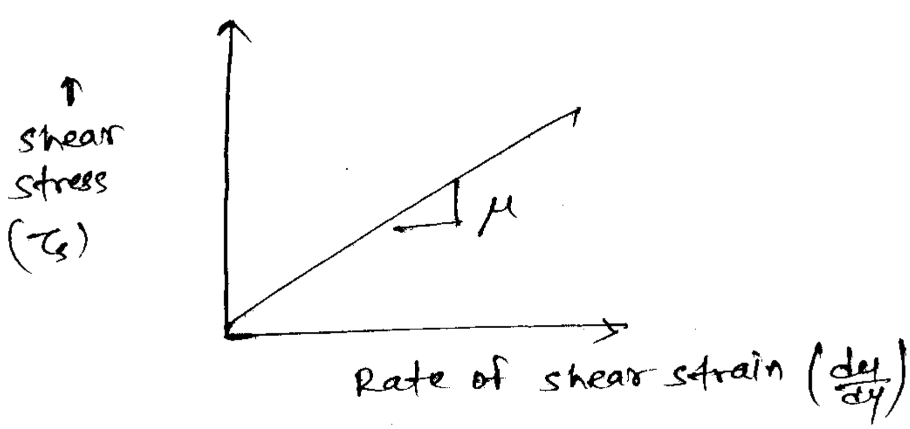
$$(1 \text{ Pa-sec} = 10 \text{ Poise})$$

* Note :- $\mu_{water} = 1 cP = 0.01$ Poise $= 0.001$ Pa-sec at 20°C

* Ratio of shear stress to velocity gradient at any point is always constant.

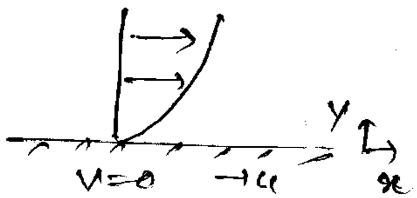
$$\boxed{\mu = \frac{\tau_s}{du/dy}} \qquad \mu \rightarrow \text{fluid property}$$

→ Viscosity is a property of the fluid and it is measured or important only when fluid is in motion. So viscosity property has no meaning when the fluid is at rest.



No slip condition :-

Velocity of the fluid relative to the solid surface is zero if the surface is at rest.
If the solid surface moves with some velocity then fluid also moves with same velocity.



at surface $y=0$

$\boxed{v=0}$

No slip condition

Kinematic viscosity :- (ν)

It is defined as the ratio between the dynamic viscosity and density of fluid.

$$\nu = \frac{\text{viscosity}}{\text{density}} = \frac{\mu}{\rho} = \text{Momentum diffusivity}$$

→ There is large variation of μ and ρ for fluid but very less variation for ν.

$$\boxed{\nu = \frac{\mu}{\rho}} \qquad \frac{N/m^2 \times sec}{kg/m^3} = \frac{m^2}{sec} \qquad \left(1N = kg \cdot \frac{m}{sec^2}\right)$$

$$\boxed{1\ \frac{cm^2}{sec} = 1\ \text{stoke}}$$

$$1\ \frac{m^2}{sec} = 10^4\ \frac{cm^2}{sec} = 10^4\ \text{stokes}$$

$$\boxed{1\ \text{stokes} = 10^{-4}\ \frac{m^2}{sec}}$$

Variation of viscosity with temperature :-

Viscous forces in the fluid is due to

(I) Cohesive force (Intermolecular force b/w the two molecules of same substance)

(II) Molecular momentum force.

1) Due to strong cohesive forces b/w the molecules, any layer in a moving fluid tries to drag the adjacent layer to move with an equal speed and thus produces the effect of viscosity.

2) The individual molecules of a fluid are continuously in motion and this thus makes a possible process of a exchange of momentum b/w different moving layer of the fluid.

*I) Viscosity of Liquids :-

In liquids the cohesive force is predominant than molecular momentum transport.

So with increase in temperature cohesive force get decreased hence viscosity gets decreased.

→ The main effect of temperature change comes not from the increase in average velocity, as in gases, but from the slight expansion of the liquid, which makes it easier for the molecules to slide past each another.

[as T ↑, μ ↓ for Liquids]

→ The viscosity of liquid is a strongly nonlinear function of the temperature but a approximation for temp. below the normal boiling point is

$$\left[\ln\mu = A + \frac{B}{T}\right] ; \left[\mu \propto \frac{1}{1+\alpha T+\beta T^2}\right]$$

α, β - constant

Imp.

* Example : viscosity of water

$\mu_{water} = 1.79$ cP at $0°C$

$\mu_{water} = 0.28$ cP at $100°C$

* $\mu_{water} = 1$ cP at $20°C$

* II) Viscosity of Gases

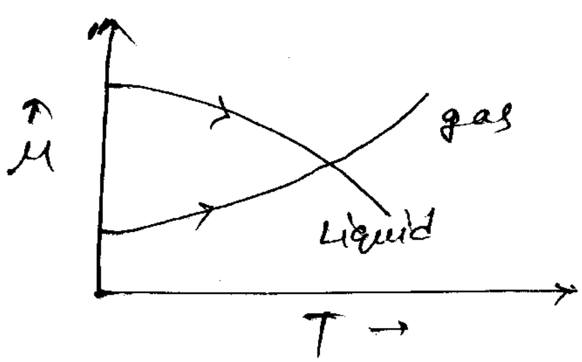
Molecular momentum transport dominates as compared to cohesive forces.

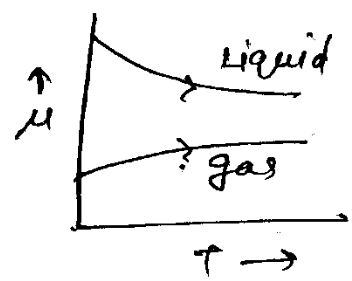
So with increase in temp. kinetic energy of molecule increase and that increase the no. of collision and hence it increase momentum transport (average velocity) and viscosity increases

[as $T\uparrow$, $\mu\uparrow$ for gases]

* from kinetic theory of gases $\mu \propto \sqrt{T}$

Where T. absolute temp.





* Gas viscosity at room temp. are generally b/w 0.005 and 0.02 cp.

$\mu_{air} = 0.018$ cp at 20°C

$\mu_{Hydrogen} = 0.009$ cp at 20°C

$\mu_{Benzene\ vapour} = 0.007$ cp at 20°C

Note:-
→ The viscosity of liquids are much greater than those of gases at the same temperature.

Example: at 20°C

$\mu_{water} = 1$ cp | $\mu_{air} = 0.018$ cp

Ideal fluid :- [Reynold No $Re = \frac{\rho v d}{\mu} = \infty$]

A fluid having a zero viscosity ($\mu = 0$) is called an "ideal fluid". and the resulting motion is called as Ideal or inviscid flow or potential flow

→ A ideal fluid flow, there is no existance of shear force because of vanishing viscosity.

Real fluid :-

All the fluids in reality have viscosity ($\mu > 0$) and hence they are termed as real fluid and their motion is known as viscous flow.

(GATE-2003)

Que-2) A Lubricant is 100 time more viscous than water would have a viscosity in pa-sec.

(i) 0.01 (ii) 0.1 ✓ (iii) 1 (iv) 10

Sol → $\mu_{Water} = 1\ cP = 0.001\ pa\text{-}sec$ at $\underline{20^\circ C}$

$\mu_{Lubricant} = 100(\mu_{water})$

$= 100(0.001)$ pa-sec

$= \underline{0.1\ pa\text{-}sec}$ → option (ii)

(GATE-2004)

Que-3) viscosity of water at 40°C lies in the range of.

(A) $1\times10^{-3} - 2\times10^{-3}$ kg/m-s (C) $1-2$ kg/m-s

(B) $0.5\times10^{-3} - 1\times10^{-3}$ kg/m-s (D) $0.5-1$ kg/m-s

Sol → We known $\mu_{water} = 1\ cP$ at 20°C

$= 0.001$ pa-sec

$\mu_{water} = 0.001\ \frac{kg}{m\text{-}s}$ at 20°C

[as T↑ μ_{liquid}↓] as temperature increase 20°C to 40°C

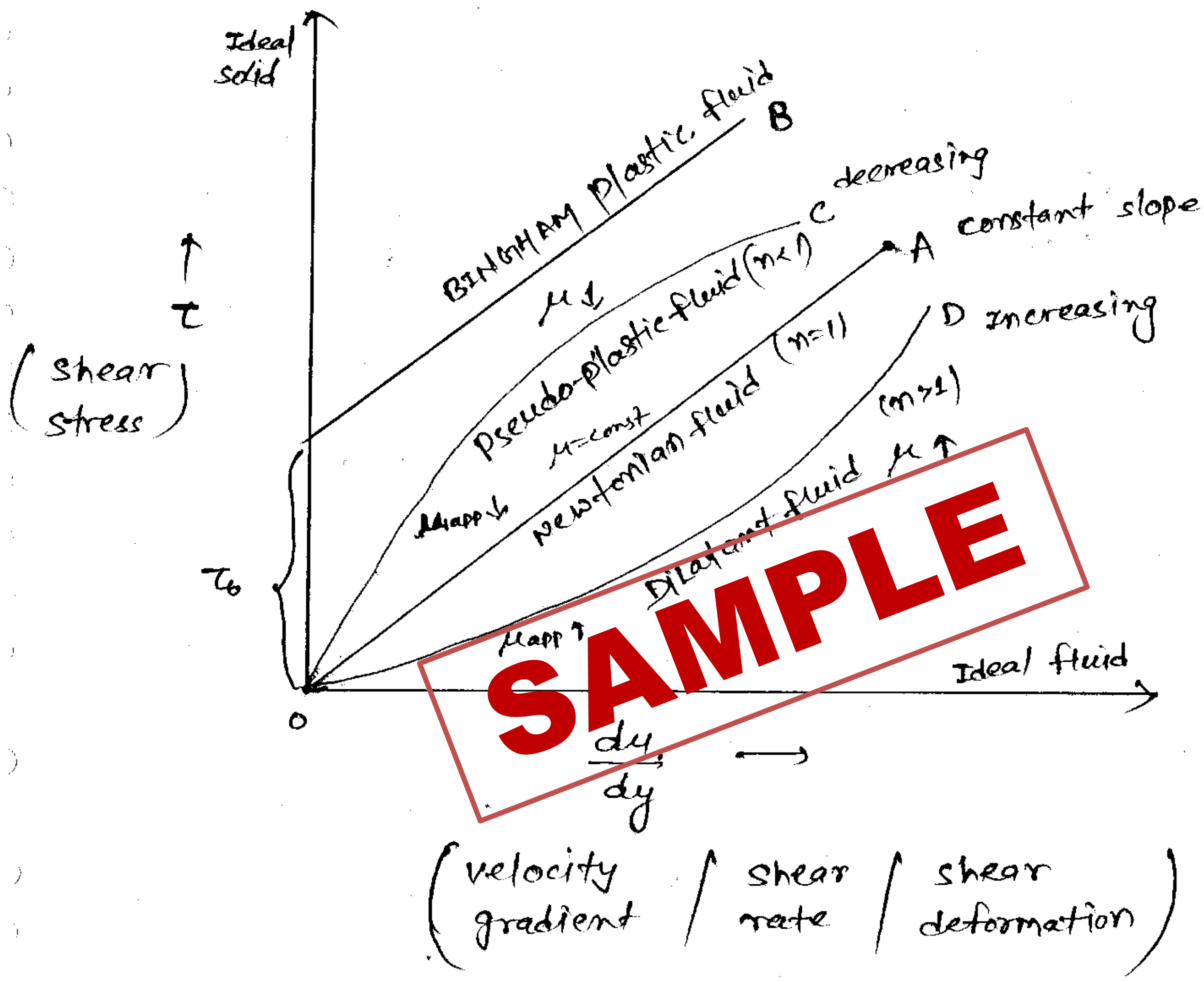
$\mu_{water} < 0.001\ kg/m\text{-}s$

$\mu_{water} < 1\times10^{-3}\ kg/m\text{-}s$

option (B) is correct

$\mu = 0.5\times10^{-3} - 1\times10^{-3}\ kg/m\text{-}s$

Rheological properties of fluid → (at const. T & P)



* There are two types of fluid :

(i) Newtonian - fluid → fluid which obeys Newton law's of viscosity

(ii) Non- Newtonian fluid (Rheological fluid) → fluid that doesn't obey Newton law of viscosity.

→ study of Non- Newtonian fluid behaviour is called Rheology.

* **Bingham plastic fluid :⇒**

$$\tau_{xy} = \tau_0 + K \frac{du}{dy}$$

where τ_0 = Threshold shear stress

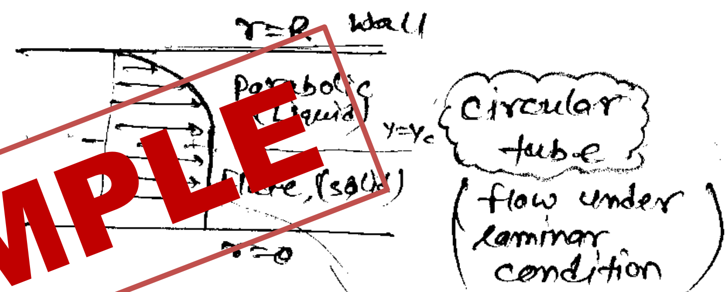
K = constant $= \mu_B$

$$\tau_{xy} = \tau_0 + \mu_B \left(\frac{du}{dy} \right)$$

; $\tau_{yx} > \tau_0$

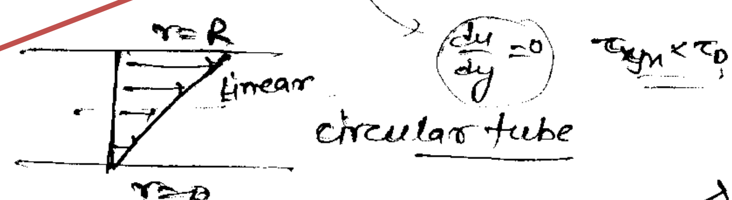
(I) **velocity profile :**

→ Parabolic near the wall and flat in middle



(II) **shear stress profile**

→ Linear



* **Dilatant & pseudo-plastic fluid ⇒**

* **power law model** or ostwald-de mode

$$\tau = K \left(\frac{du}{dy} \right)^n$$

; n = flow behaviour index,
K = flow consistency index

$$\tau = K \left| \frac{du}{dy} \right|^{n-1} \frac{du}{dy}$$

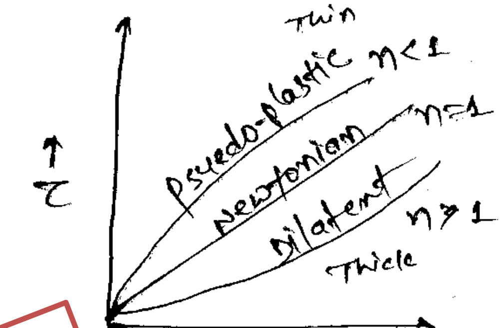
$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} = K \left| \frac{du}{dy} \right|^{n-1} = \text{Apparent viscosity}$$

* <u>Apparent viscosity</u> :-

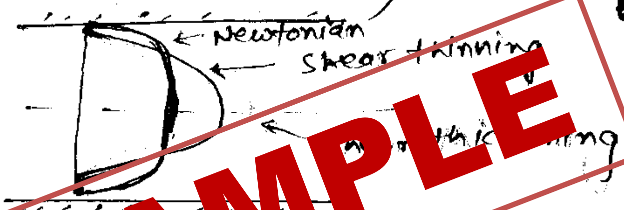
Here μ is going to change and its not property and called <u>apparent viscosity</u>.

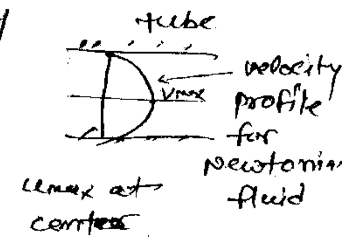
$$\mu = K \left| \frac{du}{dy} \right|^{n-1}$$

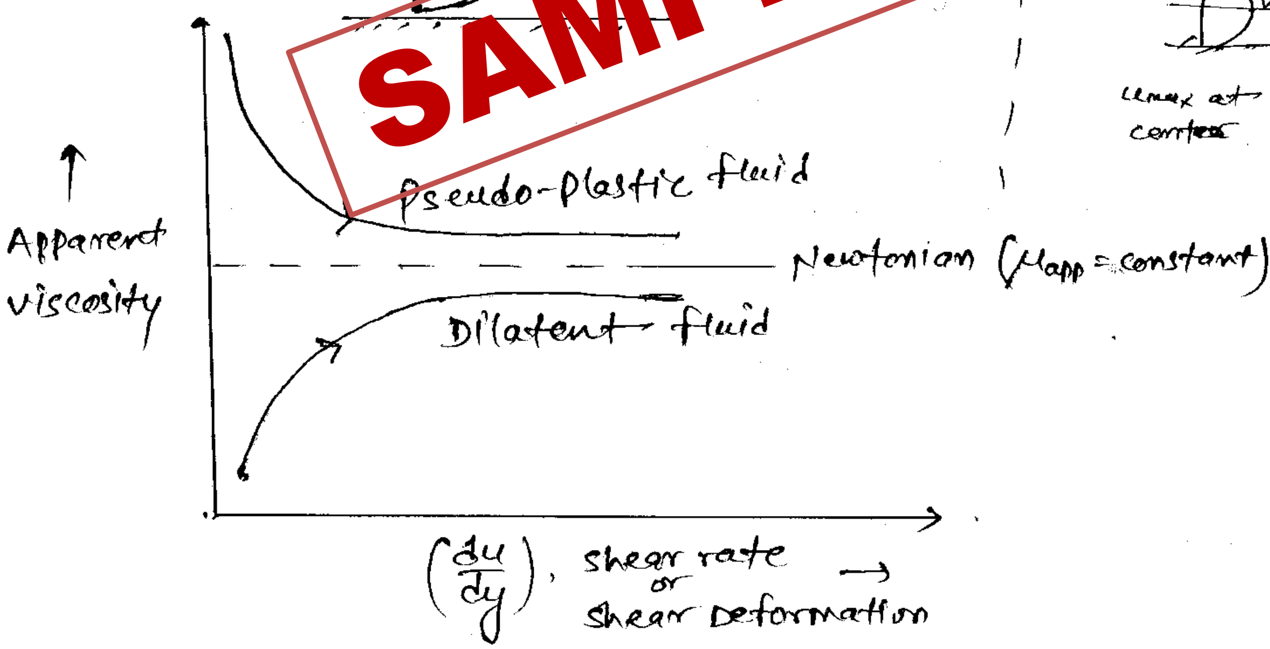
- Pseudoplastic fluid = $n < 1$
- Dilatent fluid = $n > 1$
- Newtonian fluid = $n = 0$



* velocity profile in circular tube,







* <u>Ideal fluid</u> :- fluid for which viscosity = 0
(no such fluid exist)

* <u>Ideal solid</u> :- At any shear stress there is no deformation.

Viscoelastic fluids :-

It shows both viscous and elastic properties. They exhibit elastic recovery from deformation that occur during flow, but usually only part of the deformation is recovered upon removal of the stress.

Example :- - flour dough
- certain polymer melts.

(G-2013)
Ques 7) The apparent viscosity of a fluid is given by $0.007\left(\frac{du}{dy}\right)^{0.3}$; where $\left(\frac{du}{dy}\right)$ is the velocity gradient this fluid is

(A) Bingham (B) Dilatent (C) Pseudoplastic (D) Thixotropic

Sol:- apparent viscosity $\mu = 0.007\left|\frac{du}{dy}\right|^{0.3}$

$$n-1 = 0.3$$

$$\boxed{n = 1.3} > 1$$

therefore the fluid is Dilatent (B) option

(G-2001)
Ques 8) A Bingham fluid of viscosity 10 pa-s and yield stress is 10 kpa. It shear b/w flat parallel plates separated by a distance of 10^{-3} m. The top plates is moving with a velocity of 1 m/s. the shear stress on the plate is

(A) 10 (B) 20 (C) 30 (D) 40

Sol:- $\boxed{\tau = \tau_0 + \mu\left(\frac{du}{dy}\right)} = 10\times10^3 + (10)\left(\frac{1-0}{10^{-3}-0}\right)$

$\tau_0 = 10\text{kpa} = 10\times10^3$ pa $\quad \tau = 20\times10^3$ pa

$\mu = 10$ Pa-sec

$\boxed{\tau = 20 \text{ kpa}}$ Answer option (B)

(GATE-2014)

Ques 9) Which of the following statements are correct ?

(P) For a Rheopectic fluid, the apparent viscosity increases with time under a constant applied shear stress.

(Q) For a pseudoplastic fluid, the apparent viscosity decreases with time under a constant applied shear stress.

(R) For a Bingham plastic, the apparent viscosity increases exponentially with the deformation rate.

(S) For a dilatant fluid, the apparent viscosity increases with increasing deformation rate.

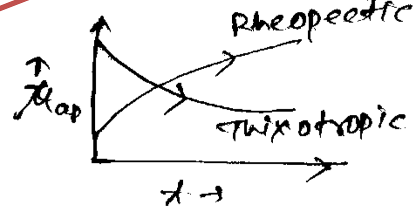
(A) P and Q only (B) Q and R only

(C) R and S only ✓(D) P and S only

Sol→ option (D) is correct P and S only

(P) Rheopectic fluid

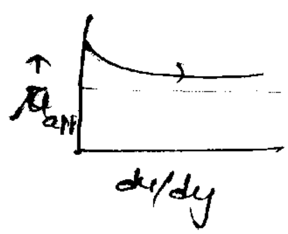
$\mu_{app} \uparrow$, w.r.t $t \uparrow$



correct

(Q) Pseudoplastic fluid

$(\mu_{app}) \downarrow$ w.r.t $(t) \uparrow$



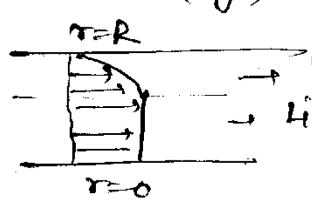
Incorrect because

$\mu_{app} \downarrow$ w.r.t $\left(\frac{du}{dy}\right)$

(R) Bingham plastic fluid

$\tau = \tau_0 + \mu\left(\frac{du}{dy}\right)$

Incorrect

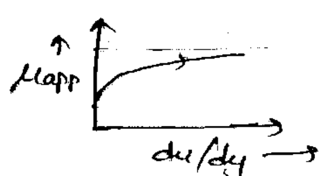


→ Parabolic near wall

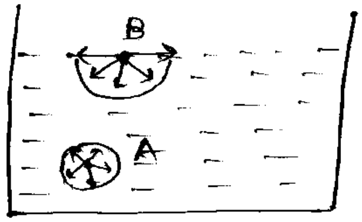
→ Linearly / flate in middle.

(S) Dilatant fluid

$\mu_{app} \uparrow, \left(\frac{du}{dy}\right) \uparrow$



Surface tension →



Molecule A is surrounded by neighbour molecules and it get attracted by neighbour molecules equally in all direction hence the net force acting on molecule A is zero. Hence A is balanced and in equilibrium but molecules B gets attracted by the neighbour molecules only in downward direction. Hence molecule 'B' is unbalanced and there is a net force acting downward,

- [Surface will be balance this molecule at the free surface, a quantum of energy expended by the fluid molecules over the surface area]

- Surface will be tension and surface feel stretched like membrane and that tensile force is called surface tension.

$$\boxed{\sigma_s = \frac{F_s}{A}} = \frac{\text{Energy}}{\text{Area}} \rightarrow \frac{J}{m^2} \rightarrow \frac{N\cdot m}{m^2}$$

$$(\sigma_s \rightarrow N/m) \qquad \boxed{\sigma_s = \frac{F_s}{L}} \; ; \; L = \text{perimeter}$$

→ The magnitude of surface tension is very small hence in all engineering calculation surface tension forces are neglected. compare to gravitational and pressure force

→ surface tension force quite significant if the boundary dimension are small.

* fluid static (fluid at rest) → weight force (Body)
→ pressure force (surface)

Pascal's Law :-

According to pascal's law intensity of pressure at a point in a fluid at rest is same in all direction. $\boxed{P_x = P_y = P_z}$

* **Hydrostatic Law :**

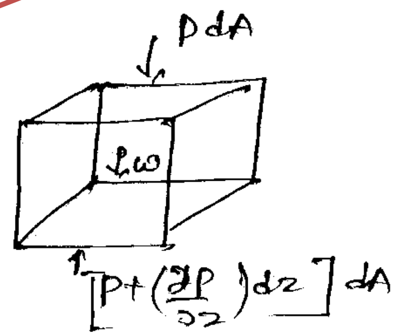
According to this law the rate of pressure increases in vertical direction is equal to weight density or specific weight of the fluid.

* consider a fluid element

W = weight force acting downward

$W = mg$

$W = \rho(dA \times dz) g$



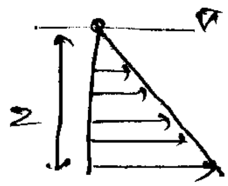
$$pdA + \rho(dAdz)g = \left[p + \left(\frac{\partial p}{\partial z}\right)dz\right]dA$$

$$\rho\, dAdz\, g = \left(\frac{\partial p}{\partial z}\right) dz\, dA$$

$$\boxed{\frac{dp}{dz} = \rho g} = \omega$$

$$\int dp = \int \rho g\, dz$$

$$\boxed{P = \rho g h + c}$$



if at $z=0$, $p=0$, then $c=0$ and p is gauge pressure

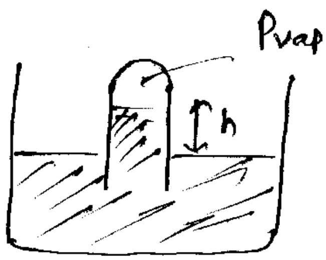
$$\boxed{P = \rho g h}, \quad (P \propto h)$$

$[P_{absolute} = P_{gauge} + 1]$

Calculation of Atmospheric pressure :-

* Barometer is used

$$P_{atm} - P_{vap} = \rho g h$$



Pvap (vapour pressure of mercury at room temp.)

* Note :- Vapour pressure of mercury is very small at room temp. and is negligible

$$P_{atm} = \rho_{Hg}\, g h$$

$\rho_{Hg} = 13600\ kg/m^3$
$\rho_w = 1000\ kg/m^3$

at 1 atm pressure $h = 760\ mm$ of Hg
$= 101.325\ kPa$
$= 101325\ Pa$

convert 760 mm Hg to mm of water

$$(13.6 \times 1000)\, 9.81 \times 760 = 1000 \times 9.81\,(h)$$

$$h = 760 \times 13.6$$

$$h = 10.33\ m \text{ of water}$$

$$h = 10.33\ m \text{ of water}$$

$$1\ atm = 760\ mm\,Hg = 10.33\ m \text{ of } H_2O$$

Manometers ⇒

manometers are used to measure pressure or pressure difference in a following medium.

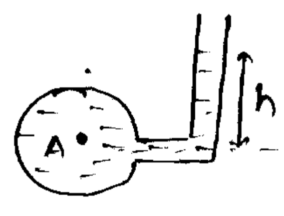
classifed into two parts
(i) simple manometers
(ii) Differential manometers

Manometer

- Simple manometer
  1) piezometer
  2) U-tube manometer
  3) single column
- Differential manometer
  1) U-tube differential manometer
  2) Inverted U-tube differential manometer

* (1) piezometer :- It is the simplest form of manometer used for measuring gauge pressure.

→ one end of this manometer is connected to the point where pressure is to be measured and other ~~point~~ end is open to the atmosphere.



→ The rise of the liquid gives the pressure head at that point A.

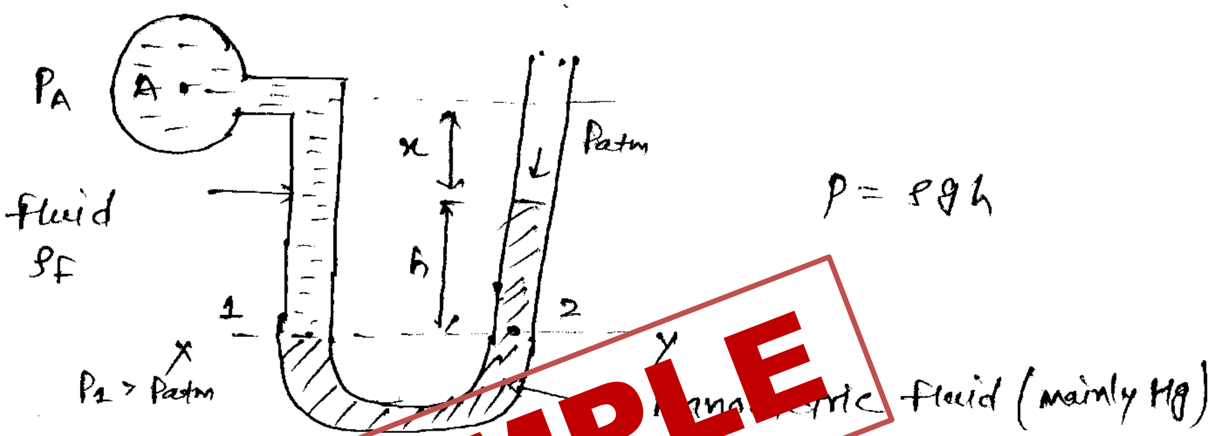
$$\boxed{P_A = \rho g h}$$: N/m^2 or Pa

* (2) U-tube manometer :-

ⓐ for Gauge pressure

ⓑ for vacuum pressure

* ⓐ For Gauge pressure :-



$P = \rho g h$

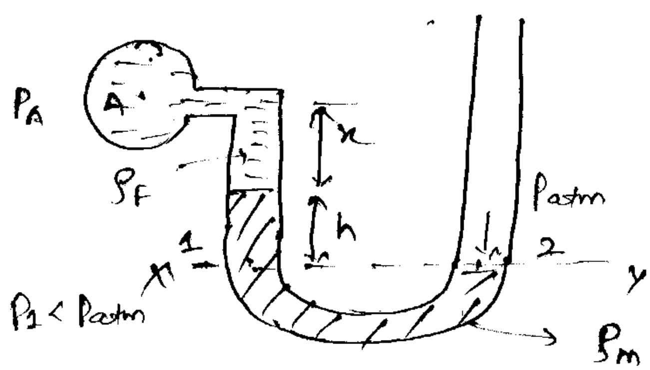
As the pressure is the same for horizontal surface.

$P_1 = P_2$

$$P_A + \rho_f g (x+h) = \rho_m g h$$

$$\left[P_A = \rho_m g h - \rho_F g (x+h) \right]$$

* ⓑ for vacuum pressure !



$P_1 = P_2$

$$P_A + \rho_F g x + \rho_m g h = 0$$

$$\left[P_A = -(\rho_m g h + \rho_F g x) \right]$$